

Effects of Gravity on the Thermodynamic Processes of Gases

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Introduction:

In classical thermodynamics the effects of gravitational force (in other words acceleration force) on the processes of gases are ignored. However when the effects of gravity are considered in the thermodynamic processes of gases many interesting results are observed. The following paper analyzes the thermodynamic processes of gases by considering the effects of gravity. And finally it analyzes the Carnot Cycle by including the effects of gravity.

Energy Analysis of a gas in a Closed System at Constant Volume when subjected to Gravity

Let us discuss the energy equilibrium in a closed system that is subjected to a gravitational field. Let us define our system and all the parameters that affect it. Consider a system that consists of a closed cylinder containing air. The cylinder is of a fixed volume and at standard temperature and pressure. The cylinder is insulated so no heat transfer takes place. The medium is kept at standard temperature and pressure. The definition of the previous parameters will allow us to study the system behavior affected by gravity only while keeping all other parameters constant.

Let us consider the cylinder displayed in figure 1 as the system we described earlier. Let us also consider that the cylinder is placed in a gravitational field g with the direction of pull pointing downwards. For mathematical analysis, let us consider that the positive direction is pointing upwards.

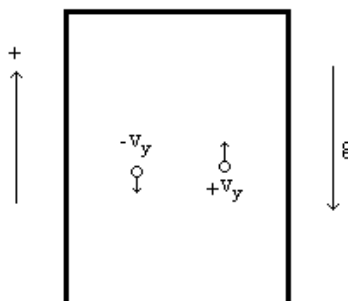


Figure 1

Now that we have defined the actual system and the mathematical parameters, let us analyze the energy distribution of the system. The gas inside the system consists of molecules that are randomly distributed in the cylinder. Each molecule has a mass and a velocity vector. The velocity vector can be defined into its' *Cartesian-coordinates*. Let us define the y-axis as the axis that is parallel to the gravitational field (as shown in figure 1).

To start the analysis we will proceed with analyzing the effect of gravity on each molecule. The velocity of a molecule, assuming no collision, can be represented as follows for a unit of time Δt .

$$\begin{aligned}
 v_{x_final} &= v_{x_initial} \\
 v_{z_final} &= v_{z_initial} \\
 v_{y_final} &= -g\Delta t + v_{y_initial}
 \end{aligned}
 \tag{1}$$

Therefore, the velocity component in the y-axis is not constant but varies with time. Gravity affects only the y-axis component (as shown in equation 1) and thus we need only to study the y-axis component of the velocities of the molecules.

Let us now look at a molecule that is colliding with the top surface of the cylinder. The time of collision with the surface is almost instantaneous (i.e. $\Delta t \approx 0$), therefore gravity does not come into the picture at the time of collision.

$$v_{y_final} = -v_{y_initial}$$

The momentum change of the molecule when it hits the top surface is:

$$\Delta p_y = p_{y_final} - p_{y_initial} = -2mv_{y_initial} \quad (2)$$

where m is the mass of the molecule,
 v initial is the speed before the collision,
 Δp is the change of momentum of the molecule.

The analysis from a molecular point of view is useless unless we can relate it to the macro world. We need to be able to determine the pressure on the top and bottom surfaces and the temperature in the system. To calculate the pressure on the top or bottom surfaces, we need to find the force exerted on either surface and divide that by the area of that surface. The force exerted by each molecule is its' momentum change after collision. The force exerted on the surface by all the molecules is the number of collisions per unit time multiplied by the change of momentum for each collision. This would only be true if all the velocities in the y-axis direction are equal, which is not the case. From Boltzmann's distribution, we can determine the distribution of all the molecular velocities and sort them out in-groups (sampling the velocity distribution). If we calculate the force due to each group of velocities we can add all the forces and calculate the total pressure on the surface.

We have already determined the momentum change for each collision, and we are left with finding out the number of collisions per unit time. Since the molecular velocities have random magnitude and direction, we can assume that half of the molecules traveling along the y-axis are in the positive direction and the other half in the negative direction. The molecules that will collide with the top surface are molecules that are at a distance of $(v_y \Delta t)$. The molecules that will strike the top surface area (A) are those molecules that are located in the volume $(A v_y \Delta t)$. The number of collisions is the multiplication of the molecules that will collide by the number of molecules per the specified volume. Since we are only considering those molecules that are moving in the positive direction, we will divide the equation by 2.

$$d = v_y \Delta t = (-g \Delta t + v_{y_initial}) \Delta t$$

$$\text{Number_of_Collisions} = \frac{1}{2} A (-g \Delta t + v_{y_initial}) \Delta t \frac{N}{V}$$

$$\frac{\text{Number_of_Collisions}}{\Delta t} = \frac{1}{2} A (-g \Delta t + v_{y_initial}) \frac{N}{V} \quad (3)$$

where A is the top surface area of the cylinder,
 N is the total number of molecules,

V is the total volume,
 $v_{y\text{-initial}}$ is the initial speed,
 g is the gravitational field constant,
 Δt is a small unit of time.

The force that the top surface exerts on the molecules is:

$$F_y = \frac{dp_y}{dt} = \frac{1}{2} A (-g\Delta t + v_{y\text{-initial}}) \frac{N}{V} (-2mv_{y\text{-initial}})$$

$$F_y = \frac{-mNA}{V} v_{y\text{-initial}}^2 + \frac{mNA}{V} v_{y\text{-initial}} g\Delta t$$
(4)

The force that the molecules exert on the top surface is the negative of the above expression. The pressure is the force divided by the area.

$$P = \frac{F_y}{A} = \frac{mN}{V} v_{y\text{-initial}}^2 - \frac{mN}{V} v_{y\text{-initial}} g\Delta t$$
(5)

Now since we already discussed that the total pressure is the summation of all the pressures of all the velocity groups, we can relate the above equation to reflect the total pressure.

$$P = \frac{mN}{V} \frac{\sum_0^n v_{y\text{-initial}}^2}{n} - \frac{mN}{V} \frac{\sum_0^n v_{y\text{-initial}}}{n} g\Delta t$$

$$P = \frac{mN}{V} v_{y\text{-average}}^2 - \frac{mN}{V} v_{y\text{-average}} g\Delta t$$

The $v_{y\text{-average}}$ is the average velocity in the y-component of the total velocity. Since the molecules have a random velocity distribution, we can assume that the average components in each of the three directions are equal.

$$v_{\text{average}}^2 = v_{x\text{-average}}^2 + v_{y\text{-average}}^2 + v_{z\text{-average}}^2$$

$$\Rightarrow v_{y\text{-average}}^2 = \frac{1}{3} v_{\text{average}}^2$$

Therefore equation 5 can be rewritten as:

$$P = \frac{mN}{V} \frac{1}{3} v_{\text{average}}^2 - \frac{mN}{V} v_{y\text{-average}} g\Delta t$$

The temperature at the top surface can be deduced from equation 6. (6)

$$PV = \frac{2}{3} nN_0 \left(\frac{1}{2} m v_{average}^2 \right) - nN_0 (mgh_{average})$$

$$\frac{3}{2} kT = \frac{1}{2} m v_{average}^2 - \frac{3}{2} mgh_{average}$$

The term $(\frac{1}{2}mv^2)$ is the kinetic energy of the system at the top surface. Therefor the energy at the top surface is: (7)

$$Top_Surface_Energy = \frac{3}{2} kT + \frac{3}{2} mgh_{average}$$

Now that we have found the pressure and temperature at the top surface, let us calculate the pressure and temperature at the bottom surface. The above analysis remains the same except for considering the molecules that are travelling in the negative direction. For that reason we will substitute a $-v_{y-initial}$ for $v_{y-initial}$. By going through the same equations above we will determine the pressure at the bottom surface.

$$P = \frac{1}{3} \frac{mN}{V} v_{average}^2 + \frac{mN}{V} v_{y-average} g \Delta t$$

$$\frac{3}{2} kT = \frac{1}{2} m v_{average}^2 + \frac{3}{2} mgh_{average} \quad (9)$$

If gravity were zero, the pressure at both surfaces would have been the same. Since we introduced the gravity factor, the bottom surface is subjected to a higher pressure than the top surface.

The energy at the bottom half is:

$$Bottom_Surface_Energy = \frac{3}{2} kT - \frac{3}{2} mgh_{average}$$

The total energy of the system is the addition of the top and bottom energies:

$$Total_Energy = Top_Surface_Energy + Bottom_Surface_Energy$$

$$Total_Energy = \frac{3}{2} kT + \frac{3}{2} mgh_{average} + \frac{3}{2} kT - \frac{3}{2} mgh_{average}$$

$$Total_Energy = 3kT$$

Therefore, the total energy of the system is constant and irrelevant of gravity. Thus the gravity changes the distribution of the gas molecules but does not change the total internal energy of the gas.

Energy Analysis of a gas in a closed system, Introduced to an Acceleration Field at Constant Volume

Let us consider the cylinder in the rotating system and analyze the distribution of the kinetic energies inside the cylinder. Since the gas is placed in a centrifuge, the molecules of the gas are subjected to a force that is equal to the mass of the molecule multiplied by the acceleration of the centrifuge. Since the process is isothermal, the temperature does not vary in the centrifuge. Consider the cylinder in the centrifuge as shown in Figure 1. To determine the pressure at a distance from the center of the centrifuge, we consider a volume with a constant area and a height Δr . The horizontal force at a unit area of distance r is the pressure P multiplied by the unit area A . The horizontal force at distance $r + dr$ has to equal the force at distance r plus the force due to the centrifuge of the gas in the unit section. Let ma be the force due to the centrifuge from each molecule of gas in the unit volume. The acceleration of the centrifuge $a = \omega^2 r$, where ω is the speed of the centrifuge, and r is the distance from the center of the centrifuge. Let N/V be the number of molecules per volume in the cylinder and Adr be the volume of the unit section. The total number of molecules in the unit section is $NAdr/V$. Therefore $AP_{r+dr} - AP_r = AdP = -NmaAdr/V$. Now from the equation of ideal gas, $P = NkT/V$. Since N and P are directly related we can solve for P or N , say N .

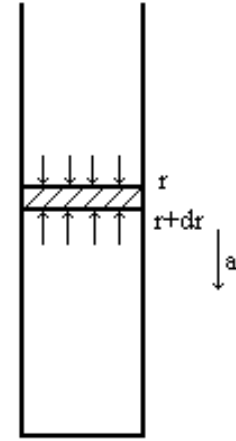


Figure 1.

$$dP = -\frac{N}{V} m a dr$$

$$dN = -N \frac{ma}{kT} dr \quad (1)$$

where m is the mass of a molecule of the gas,
 a is the acceleration of the centrifuge,
 k is Boltzmann's constant,
 T is the Temperature of the gas,
 r is the distance from the center of the centrifuge,
 N is the number of molecules of the gas.

If the speed of the centrifuge were zero, then the number of molecules per layer would be equal to the number of molecules calculated using the ideal gas equation. The distribution of the molecules in the system does not alter the total number of molecules of the gas. Therefore, we can define a layer in the system where the number of molecules at that layer is equal to the

average number of molecules, or the number of molecules with out the centrifugal affect. Let us call the number of molecules at that layer N_c . The layer is located at a distance r_c from the center of the centrifuge. The values of N_c and r_c change as the volume change. The value of N_c can be determined by using the ideal gas equation and we can solve for the value of r_c . Integrating equation (1) from the layer at distance r_c to any layer at distance r yields:

$$\int_{N_c}^N \frac{dN}{N} = \int_{r_c}^r \frac{-ma}{kT} dr$$

$$N = N_c e^{\frac{-ma(r-r_c)}{kT}} \quad (2)$$

Since the pressure is directly proportional to the number of molecules, it could be shown also that :

$$P = P_c e^{\frac{ma(r-r_c)}{kT}}$$

To calculate r_c :

$$P_c = P_0 e^{\frac{mar_c}{kT}}$$

$$P_h = P_0 e^{\frac{maH}{kT}}$$

where P_h is the pressure at a height H ,

P_0 is the pressure at the bottom of the enclosure $h = 0$.

The difference between P_h and P_0 is simply the weight of the gas divided by the area of the cylinder. The pressure P_c is the pressure determined from equation (3).

$$P_0 - P_h = \frac{Nma}{A}$$

$$\Rightarrow \frac{Nmg}{A} = P_0 - P_0 e^{\frac{maH}{kT}}$$

$$\Rightarrow P_0 = \frac{\frac{Nma}{A}}{1 - e^{\frac{maH}{kT}}}$$

Relating the term found above to P_c we get:

$$P_c = \frac{Nma}{A} e^{-\frac{mar_c}{kT}}$$

$$\frac{NkT}{AH} = \frac{Nma}{A} e^{-\frac{mgh_c}{kT}}$$

$$e^{-\frac{mar_c}{kT}} = \frac{kT}{maH} \left(1 - e^{-\frac{maH}{kT}} \right) \quad (3)$$

$$e^{\frac{mar_c}{kT}} = \frac{maH}{kT} \frac{1}{1 - e^{-\frac{maH}{kT}}}$$

Using equation (2), we can plot a graph of the distribution of molecules in the cylinder vs. the height of the cylinder.

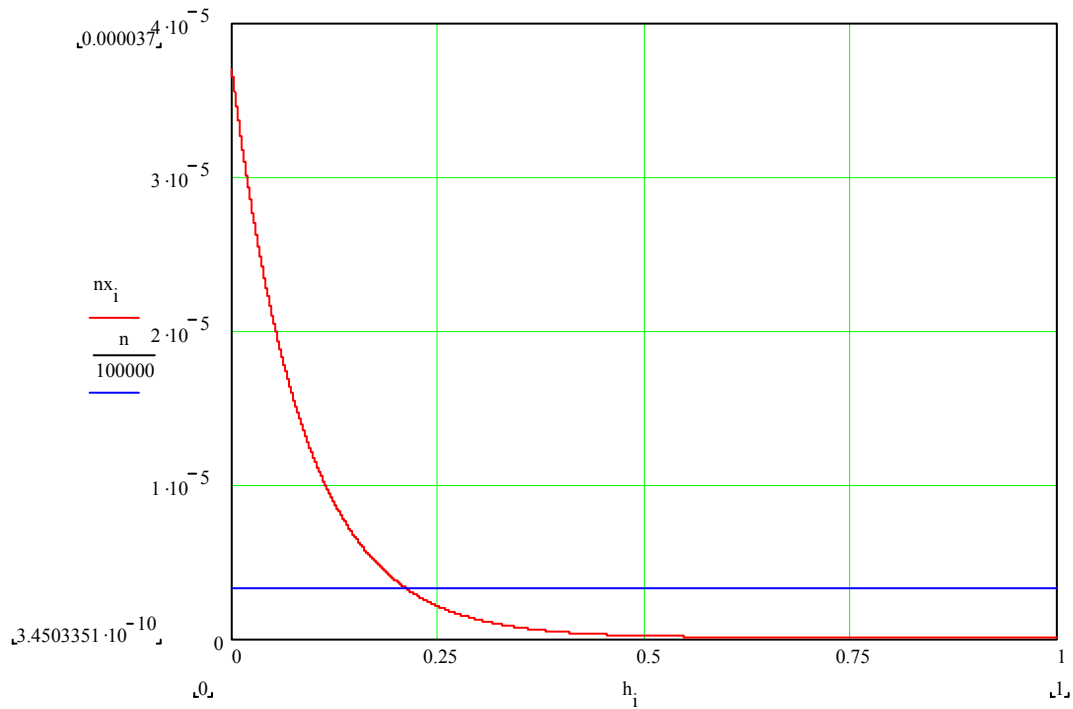


Figure 2. The red line represents the distribution of the moles in the cylinder for an acceleration of $1,000,000\text{m/s}^2$. The blue line is the distribution of the moles in a zero acceleration field. The point of intersection of both lines is the distance r_c from the bottom of the cylinder.

In order to determine if the energy is conserved when the system is subjected to an acceleration field, we need to calculate the energies of both the compressed part of the cylinder and the expanded part of the cylinder. Since the molecules of the gas in the system are redistributed, we would get an increase in the number of molecules at the bottom of the cylinder and a decrease in the number of the molecules at the top of the cylinder. The point at which the number of molecules is equal to a normal distribution of molecules is the dividing layer in the cylinder. We will designate that point as r_c , which is also the intersection of the two lines in figure 2. Since the temperature is the same throughout the cylinder, it implies that the v_{rms} of the molecules of the system is the same also. Therefore, the kinetic energy of the molecules KE at any layer is equal to the number of molecules in that layer multiplied by the mass of each molecule and the root-mean square velocity of the molecules ($KE = \frac{1}{2} Nm v_{rms}^2$).

The total kinetic energy of the molecules in the cylinder is the summation of all the kinetic energies in each layer. We can divide the cylinder into two parts called the compression and the expansion part. The compression parts are the layers from the bottom of the cylinder to the layer called r_c . The number of molecules in the compression part increases as the value of the acceleration field increases. The expansion parts are the layers from the top of the cylinder to the layer called r_c . The number of molecules in the expansion part decreases as the value of the acceleration field increases. In a normal distribution, the total kinetic energy of the cylinder is divided into two equal parts the bottom half (which could be called compression part) and the top half (which could be called expansion part).

In order to study the energy distribution when the cylinder is in an acceleration field, the kinetic energy deviation from normal kinetic energy distribution in the compression part is calculated. Similarly the kinetic energy deviation from normal kinetic energy distribution in the expansion part is calculated. If the deviations from the norm from both the expansion and compression parts are equal, then the total kinetic energy of the molecules is conserved.

Analysis of the Carnot's Cycle considering Gravitation:

When the effects of gravitation are considered in the analysis of the Carnot's cycle it is called the Amin Cycle. Let us analyze the Amin Cycle while considering the effects of gravitational forces on the gas. The Amin Cycle uses an ideal gas as a working substance and is shown as a physical process in Figure 1, and on a Temperature-Entropy diagram in Figure 2. Figure 2 is only a partial explanation of the Amin cycle, the more precise diagram is shown in Figure 3 and discussed in more detail towards the end of the paper. The cycle comprises the following steps:

- 1) The gas expands isothermally at a temperature T in a low gravitational field g_2 absorbing heat Q_1 and producing W_1 work.
- 2) The enclosure containing the gas is moved to a higher gravitational field g_1 producing work W_2 and rejecting heat Q_2 .
- 3) The gas is compressed isothermally at a temperature T in a high gravitational field g_1 producing heat Q_3 and requiring W_3 work.
- 4) The enclosure containing the gas is moved back to the low gravitational field g_2 requiring work W_4 and absorbing heat Q_4 .

We will assume that the enclosure holding the gas is of a constant area to simplify the mathematical equations derived from here on. The constant area assumption is not required and the process can work with any volume change.

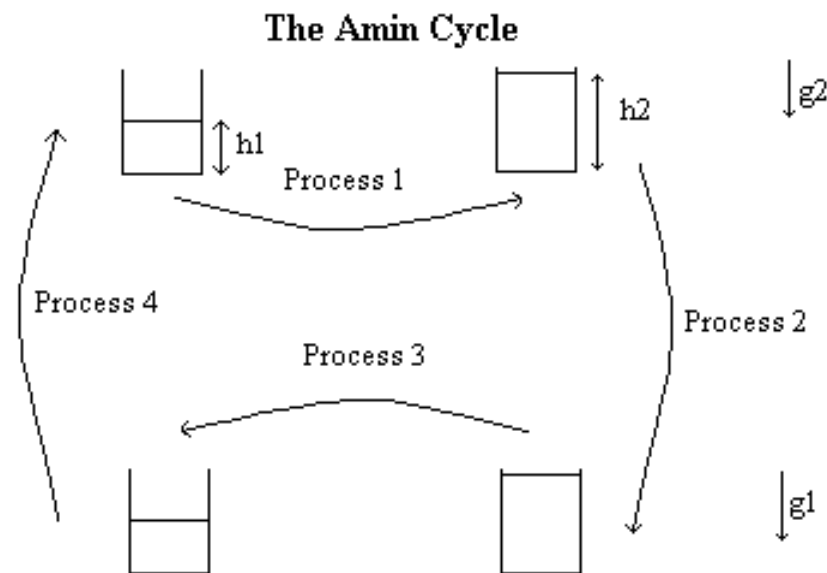


Figure 1, physical processes of the Amin Cycle.

Let us analyze each process separately. **Process 1** is the isothermal expansion from height h_1 to height h_2 in a low gravitational field g_2 at a temperature T .

$$dQ = dW + dU + dPE \quad (1)$$

Since the process is isothermal, $dU = 0$.

$$dW = PdV$$

$$dW = PAdh \quad (2)$$

Since we are considering an ideal gas, we can refer to the ideal gas equation:

$$P = \frac{NkT}{V} \quad (3)$$

where N is the number of molecules in the gas,
 k is Boltzmann's constant,
 T is the temperature of the gas,
 V is the volume of the gas.

Since the gas is subjected to a gravitational field, a density distribution is created. The distribution is given by Boltzmann's Law of gravitational distribution. The average temperature does not vary in a gravitational field. Consider an enclosure in a gravity field as shown in Figure 2. To determine the pressure at a height we consider a volume with a constant area and a height Δh . The vertical force at a unit area of height h is the pressure P multiplied by the unit area A . The vertical force at height $h + dh$ would be the same as the force at height h without gravity. Now since gravity is introduced, the force at height h has to equal the force at height $h + dh$ plus the weight of the gas in the unit section. Let mg be the force due to weight from each molecule of gas in the unit volume. Let N/V be the number of molecules per volume in the cylinder and Adh be the volume of the unit section. The total number of molecules in the unit section is $NAdh/V$. Therefore $AP_{h+dh} - AP_h = AdP = -NmgAdh/V$. Now from the equation of ideal gas, $P = NkT/V$. Since N and P are directly related we can solve for P or N , say P .

$$dP = -\frac{N}{V}mgdh$$

$$dP = -P\frac{mg}{kT}dh$$

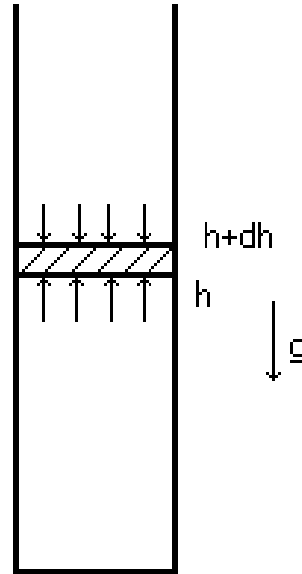


Figure 2, . The pressure at height h must exceed that at $h + dh$ by the weight of the intervening gas.

(4)

where m is the mass of a molecule of the gas,
 g is the gravitational field affecting the gas,

If gravity were zero, then the pressure differential is zero and equation (3) would be used to calculate the pressure at any point. The distribution of the pressures in the system does not alter the value of the average pressure of the gas. Since no volume or average temperature change occur in the presence of a gravitational field, the average pressure remains the same. Therefore, we can define a layer in the system where the pressure at that layer is equal to the average pressure, or the pressure with out gravity. Let us call the pressure at that layer P_c . The layer is located at a height h_c from the bottom of the enclosure. The values of P_c and h_c change as the volume or average temperature change. The value of P_c can be determined by using the ideal gas equation (3), and we can solve for the value of h_c . Integrating equation (4) from the layer at height h_c to any layer at height h yields:

$$\int_{P_c}^P \frac{dP}{P} = \int_{h_c}^h \frac{-mg}{kT} dh$$

$$P = P_c e^{-\frac{mg(h-h_c)}{kT}} \quad (5)$$

To calculate h_c :

$$P_c = P_0 e^{-\frac{mgh_c}{kT}}$$

$$P_h = P_0 e^{-\frac{mgh}{kT}}$$

where P_h is the pressure at a height h ,
 P_0 is the pressure at the bottom of the enclosure $h = 0$.

The difference between P_h and P_0 is simply the weight of the gas divided by the area of the cylinder. The pressure P_c is the pressure determined from equation (3).

$$P_0 - P_h = \frac{Nmg}{A}$$

$$\Rightarrow \frac{Nmg}{A} = P_0 - P_0 e^{-\frac{mgh}{kT}}$$

$$\Rightarrow P_0 = \frac{\frac{Nmg}{A}}{1 - e^{-\frac{mgh}{kT}}}$$

Relating the term found above to P_c we get:

$$\begin{aligned}
 P_c &= \frac{Nmg}{A} \frac{e^{-\frac{mgh_c}{kT}}}{1 - e^{-\frac{mgh}{kT}}} \\
 \frac{NkT}{Ah} &= \frac{Nmg}{A} \frac{e^{-\frac{mgh_c}{kT}}}{1 - e^{-\frac{mgh}{kT}}} \\
 e^{-\frac{mgh_c}{kT}} &= \frac{kT}{mgh} \left(1 - e^{-\frac{mgh}{kT}} \right) \\
 e^{\frac{mgh_c}{kT}} &= \frac{mgh}{kT} \frac{1}{1 - e^{-\frac{mgh}{kT}}} \tag{6}
 \end{aligned}$$

Substituting the term found in equation (6) for h_c into equation (5) and using the value of P_c as determined by equation (3), we get:

$$\begin{aligned}
 P &= \frac{NkT}{Ah} e^{-\frac{mgh}{kT}} \left(\frac{mgh}{kT} \frac{1}{1 - e^{-\frac{mgh}{kT}}} \right) \\
 P &= \frac{Nmg}{A} \frac{e^{-\frac{mgh}{kT}}}{1 - e^{-\frac{mgh}{kT}}} \tag{7}
 \end{aligned}$$

Substituting the term P from equation (7) into equation (2) and replacing the term g by the term g_2 to represent the gravitational field at process 1 we can determine the work done by the gas when expanding from h_1 to h_2 .

$$\begin{aligned}
 dW &= \frac{Nmg_2}{A} \frac{e^{-\frac{mg_2 h}{kT}}}{1 - e^{-\frac{mg_2 h}{kT}}} Adh \\
 \int_1^2 dW &= \int_{h_1}^{h_2} Nmg_2 \frac{e^{-\frac{mg_2 h}{kT}}}{1 - e^{-\frac{mg_2 h}{kT}}} dh \\
 W_1 &= NkT \ln \left(1 - e^{-\frac{mg_2 h}{kT}} \right) \Big|_{h_1}^{h_2}
 \end{aligned}$$

$$W_1 = NkTLn \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) \quad (8)$$

where N is the number of molecules in the system,
k is the Boltzmann's constant,
T is the temperature of the system,
m is the mass of a molecule,
g₂ is the gravitational field,
h₁ is the height of the enclosure before expansion,
h₂ is the height of the enclosure after expansion.

If g₂ = 0 then the solution for equation (8) becomes undetermined (0/0). Since the result of the limit is undeterminable, we apply L'Hôpital's Rule by taking the kth derivative of the numerator and the denominator until we can get a definite value.

$$\begin{aligned}
W_1 &= \lim_{g_2 \rightarrow 0} \left(NkTLn \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) \right) \\
W_1 &= \lim_{g_2 \rightarrow 0} \left(NkTLn \left(\frac{\frac{d}{dg_2} \left(1 - e^{-\frac{mg_2 h_2}{kT}} \right)}{\frac{d}{dg_2} \left(1 - e^{-\frac{mg_2 h_1}{kT}} \right)} \right) \right) \\
W_1 &= \lim_{g_2 \rightarrow 0} \left(NkTLn \left(\frac{\frac{mh_2}{kT} e^{-\frac{mg_2 h_2}{kT}}}{\frac{mh_1}{kT} e^{-\frac{mg_2 h_1}{kT}}} \right) \right) \\
W_1 &= NkTLn \left(\frac{h_2}{h_1} \right) \quad (9)
\end{aligned}$$

Equation (9) describes the work done by the gas in an isothermal expansion process in a zero gravitational field. This is the equation for work done by an isothermal process in classical thermodynamics.

The potential energy difference of the system can be determined by computing the potential energy of the gas after expansion minus the potential energy of the gas before expansion. The potential energy of the gas is the sum of the potential energies of all the molecules.

$$PE = \sum mgh$$

$$PE = Nmg \frac{\sum h}{N}$$

The $\sum h/N$ is just the average height of the distribution of the molecules. Since the number of the molecules per layer is directly proportional to the pressure on that layer, the distribution of N can be determined from equation (4) by solving for N instead of P . From the subsequent derivation of the distribution of P and the determination of a layer at a height h_c , we can relate the distribution of molecules to the same height h_c . Therefore, the $\sum h/N$ is no other than h_c . From equation (6) we can determine a value for h_c .

$$h_c = \frac{kT}{mg} \text{Ln} \left(\frac{mg}{kT} h \left(\frac{1}{1 - e^{-\frac{mgh}{kT}}} \right) \right)$$

$$\Rightarrow PE = NkT \text{Ln} \left(\frac{mg}{kT} h \left(\frac{1}{1 - e^{-\frac{mgh}{kT}}} \right) \right) \quad (10)$$

The difference in potential energy when expanding the volume from h_1 to h_2 is:

$$\Delta PE = NkT \text{Ln} \left(\frac{mg}{kT} h_2 \left(\frac{1}{1 - e^{-\frac{mgh_2}{kT}}} \right) \right) - NkT \text{Ln} \left(\frac{mg}{kT} h_1 \left(\frac{1}{1 - e^{-\frac{mgh_1}{kT}}} \right) \right)$$

$$\Delta PE = NkT \text{Ln} \left(\frac{h_2}{h_1} \left(\frac{1 - e^{-\frac{mgh_1}{kT}}}{1 - e^{-\frac{mgh_2}{kT}}} \right) \right)$$

$$\Delta PE = NkT \text{Ln} \left(\frac{h_2}{h_1} \right) - NkT \text{Ln} \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) \quad (11)$$

The heat absorbed by the system during expansion Q_1 is calculated from equation (1).

$$\begin{aligned}
\Delta Q &= Q_1 = W_1 + \Delta PE \\
Q_1 &= NkTLn \left(\frac{1 - e^{-\frac{mgh_2}{kT}}}{1 - e^{-\frac{mgh_1}{kT}}} \right) + NkTLn \left(\frac{h_2}{h_1} \right) - NkTLn \left(\frac{1 - e^{-\frac{mgh_2}{kT}}}{1 - e^{-\frac{mgh_1}{kT}}} \right) \\
Q_1 &= NkTLn \left(\frac{h_2}{h_1} \right) \tag{12}
\end{aligned}$$

Process 2 is bringing the entire enclosure to a higher gravitational field. Since this is a theoretical analysis, we can assume that the change of gravity can occur without subjecting the cylinder to any other process. Now let us refer to equation (1) and review the energy changes inside the enclosure. Since there were no processes done on the gas by the change of gravity, i.e. no compression or expansion, $dW = 0$. The cycle is done isothermally which means $dU = 0$. Therefore, equation (1) reduces to:

$$\begin{aligned}
dQ &= dPE \\
Q_2 &= \Delta PE \tag{13}
\end{aligned}$$

The potential energy difference can be determined by subtracting the potential energy of the gas at the low gravity potential g_1 from the potential energy of the gas at the high gravity potential g_2 .

$$\begin{aligned}
Q_2 &= Nmg_2 h_{c2} - Nmg_1 h_{c1} \\
Q_2 &= Nmg_2 \left(\frac{kT}{mg_2} Ln \left(\frac{mg_2}{kT} \left(\frac{1}{1 - e^{-\frac{mg_2 h_2}{kT}}} \right) \right) \right) \\
&\quad - Nmg_1 \left(\frac{kT}{mg_1} Ln \left(\frac{mg_1}{kT} \left(\frac{1}{1 - e^{-\frac{mg_1 h_2}{kT}}} \right) \right) \right) \\
Q_2 &= NkTLn \left(\frac{\frac{mg_2}{kT} \left(\frac{1}{1 - e^{-\frac{mg_2 h_2}{kT}}} \right)}{\frac{mg_1}{kT} \left(\frac{1}{1 - e^{-\frac{mg_1 h_2}{kT}}} \right)} \right) \\
Q_2 &= NkTLn \left(\frac{g_2 \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_2}{kT}}} \right)}{g_1 \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_2}{kT}}} \right)} \right) \tag{14}
\end{aligned}$$

where N is the total number of molecules,
 k is Boltzmann's constant,
 T is the temperature of the system,
 m is the mass of a molecule,
 g_1 is the high gravity potential,
 g_2 is the low gravity potential,
 h_2 is the height of the gas related to the volume after expansion,
 Q_2 is the heat rejected from the system.

Process 3 is the compression in a high gravitational field. The compression is done isothermally bringing the volume of the cylinder back to its original volume, i.e. the volume before the expansion. The derivation of the formulas is similar to the expansion process. The gravitational field g_2 is replaced with the gravitational field g_1 . The differential formulas are integrated from h_2 to h_1 . Following the derivation for the work for expansion found in equation (8), we can determine the work done for compression.

$$W_3 = -NkTLn \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}} \right) \quad (15)$$

The potential energy difference can also be determined for the compression similarly to the expansion process. The derivation leading up to equation (11) can be substituted for the high gravitational field, and the compression.

$$\Delta PE = -NkTLn \left(\frac{h_2}{h_1} \right) + NkTLn \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}} \right) \quad (16)$$

Therefore the heat rejected by the gas during the isothermal compression stage is $dQ = dW + dU + dPE$, which is:

$$Q_3 = -NkTLn \left(\frac{h_2}{h_1} \right) \quad (17)$$

Process 4 is to return the entire enclosure to the low gravitational field g_2 . This process is identical to the earlier process of bringing the entire enclosure to a high gravitational field g_1 . The heat absorbed by the system can also be derived similarly to equation (14).

$$Q_4 = -NkTLn \left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) \right) \quad (18)$$

Let us now examine the net work done by or on the system by adding the work at each stage. The net work done is the work due to expansion plus the work due to gravity change plus the work done due to compression plus the work done due to gravity change.

$$\begin{aligned}
 Net_Work &= W_1 + W_2 + W_3 + W_4 \\
 Net_Work &= NkT \ln \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) + 0 \\
 &- 0 - NkT \ln \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}} \right) \\
 Net_Work &= NkT \ln \left(\left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) \left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_1 h_2}{kT}}} \right) \right) \tag{19}
 \end{aligned}$$

Let us also look at the heat absorbed or rejected by the system for the entire cycle.

$$\begin{aligned}
 Total_Heat &= Q_1 + Q_2 + Q_3 + Q_4 \\
 Total_Heat &= NkT \ln \left(\frac{h_2}{h_1} \right) + NkT \ln \left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_2}{kT}}} \right) \right) \\
 &- NkT \ln \left(\frac{h_2}{h_1} \right) - NkT \ln \left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) \right) \\
 Total_Heat &= NkT \ln \left(\left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}} \right) \left(\frac{1 - e^{-\frac{mg_2 h_1}{kT}}}{1 - e^{-\frac{mg_2 h_2}{kT}}} \right) \right) \tag{20}
 \end{aligned}$$

The first rule of thermodynamics states that the energy for the entire cycle is conserved. That is proven by comparing equations (19) and (20). Thus we have converted heat into work without a temperature differential but with only a potential energy differential.

The Amin Cycle can be represented in comparison to the Carnot's Cycle as shown in Figure 3. The main difference between the Amin Cycle and the Carnot's Cycle is that in the Amin Cycle, the compression and expansion processes are done at different magnitudes of

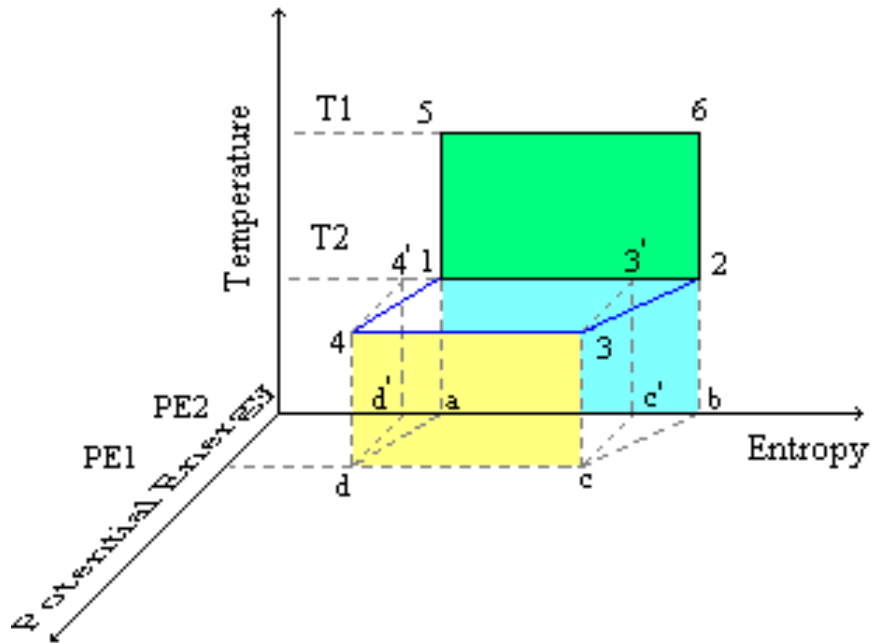


Figure 3, is the Temperature-Entropy Diagram, a true representation of the Amin Cycle. The addition of the third dimension, the potential energy (in other words kinetic energy), better describes the increase and decrease in kinetic energy with respect to entropy. The points 3' and 4', which are points 3 and 4 in figure (a), are the projection of points 3 and 4 respectively on the Temperature-Entropy plane. The area (1-2-b-a) is the heat input, and the area (4-3-c-d) is the heat output. PE2 is the lower kinetic energy value and PE1 is the higher kinetic energy value. Area (4-3-c-d) is smaller than Area (1-2-b-a) and the difference between these two areas is the heat converted to power. The work done to increase and decrease the kinetic energy does not depend on the path but only on the end points. [eqn. (22) & (24)].

Potential Energy, while in the Carnot's Cycle, the processes are done at the same magnitude of Potential Energy. The introduction of the third dimension in Figure 3 is due to the change of potential energy in a process. Carnot Cycle operates at constant potential energy. However, if we introduce a variable potential energy, the process no longer occupies a two dimensional picture.

In the Carnot's Cycle, all the thermodynamic processes are done at the same magnitude of potential energy but require a temperature differential to convert heat to power. In the Amin Cycle, the thermodynamic processes require either a potential energy differential or a temperature differential, or a combination of both. The Amin Cycle can convert heat to power solely due to a gravitational differential.

The efficiency of the cycle is the net work divided by the heat input. The efficiency can also be expressed as $(1 - \text{heat out}/\text{heat in})$. Therefore let us calculate the heat out and the heat in.

$$Q_{-out} = Q_3 + Q_2 \quad (21)$$

$$Q_{-out} = -NkT \ln \left(\frac{h_2}{h_1} \right) + NkT \ln \left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_2}{kT}}} \right) \right)$$

$$Q_{-out} = -NkT \ln \left(\frac{h_2 g_1}{h_1 g_2} \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_2}{kT}}} \right) \right) \quad (22)$$

The negative sign in equation (22) is just a mathematical representation of the direction of heat flow. Since we indicate that the heat is rejected, we can use the absolute value of equation (24).

$$Q_{-in} = Q_1 + Q_4 \quad (23)$$

$$Q_{-in} = NkT \ln \left(\frac{h_2}{h_1} \right) - NkT \ln \left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}} \right) \right)$$

$$Q_{-in} = NkT \ln \left(\frac{h_2 g_1}{h_1 g_2} \left(\frac{1 - e^{-\frac{mg_2 h_1}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}} \right) \right) \quad (24)$$

The efficiency is:

$$\begin{aligned}
 \varepsilon &= 1 - \frac{Q_{out}}{Q_{in}} \\
 \varepsilon &= 1 - \frac{NkT \ln \left(\frac{h_2 g_1}{h_1 g_2} \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_2}{kT}}} \right) \right)}{NkT \ln \left(\frac{h_2 g_1}{h_1 g_2} \left(\frac{1 - e^{-\frac{mg_2 h_1}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}} \right) \right)} \\
 \varepsilon &= 1 - \frac{\ln \left(\frac{h_2 g_1}{h_1 g_2} \left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_2}{kT}}} \right) \right)}{\ln \left(\frac{h_2 g_1}{h_1 g_2} \left(\frac{1 - e^{-\frac{mg_2 h_1}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}} \right) \right)} \tag{25}
 \end{aligned}$$

Let us note that if the magnitudes of the gravitational fields are equal, i.e. $g_1 = g_2$, the term Q_2 in equation (21) and the term Q_4 in equation (23) would be equal to zero. Therefore the term $Q_{out}/Q_{in} = 1$, and the efficiency = 0.

The heat is still represented as the area underneath the temperature-entropy curve. The heat can also be represented as the temperature multiplied by the difference in entropy for a reversible process.

Let us check equation (25) again by applying the Law of Conservation of Energy.

Conservation of Energy Test:

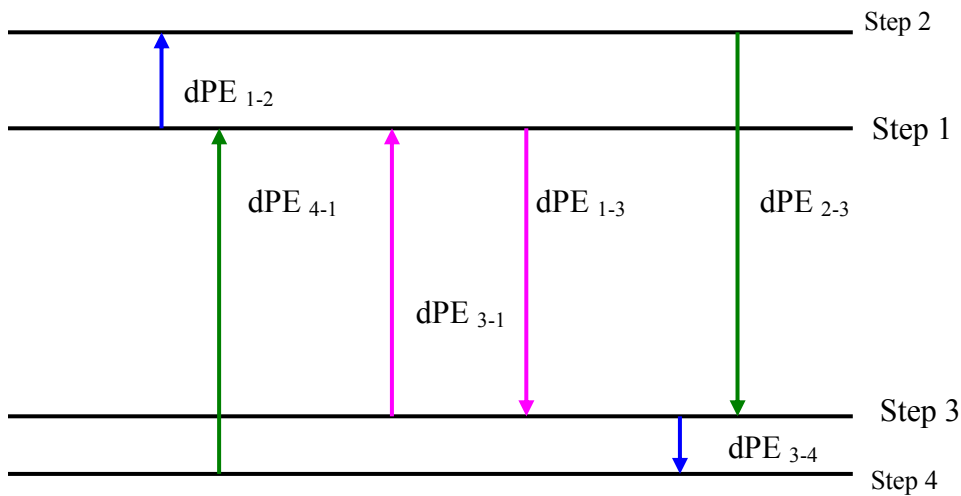


Figure 4, Representation of the potential energies in the Amin Cycle.

1. CONSERVATION OF ENERGY TEST:

According to the law of conservation of energy, the summation of the potential energies of the cycle $dPE_{1-2} + dPE_{2-3} + dPE_{3-4} + dPE_{4-1} = 0$ because our start and end points are the same.

$$dPE_{3-1} = dPE_{4-1} - dPE_{3-4}$$

$$dPE_{1-3} = dPE_{2-3} - dPE_{1-2}$$

$$\text{but } dPE_{3-1} + dPE_{1-3} = 0$$

$$\Rightarrow dPE_{2-3} - dPE_{3-4} + dPE_{4-1} - dPE_{1-2} = 0$$

$$\& dPE_{2-3} + dPE_{3-4} + dPE_{4-1} + dPE_{1-2} = 0$$

$$\Rightarrow 2 dPE_{2-3} + 2 dPE_{4-1} = 0$$

$$\Rightarrow dPE_{2-3} = - dPE_{4-1}$$

$$\& dPE_{3-4} = - dPE_{1-2}$$

When Net work = 0

dPE_{3-4} has to equal $-dPE_{1-2}$.

Conclusion:

Equation (25) gives the quantity of heat converted to power by the Amin Cycle for a specific magnitude of potential energy difference. It also indicates that heat can be converted to

power by the Amin Cycle even without a temperature difference as long as there is a potential energy difference.

Most matter above absolute zero has heat energy in it. Thus we live in an ocean of energy but we cannot utilize it, as we did not have the knowledge to convert this heat energy to power. But with the Amin Cycle we can convert heat at any temperature to power and this makes all the heat energy in the universe available.

According to Carnot's Cycle, the most efficient conversion of heat to power between two temperature differences is achieved if the entropy of the system remains constant. Equation (28) indicates that when heat is converted to power by the Amin Cycle, a net entropy reduction is achieved. Thus changes in entropy are reversible like any other physical process.

Carnot Cycle considering the effects of Acceleration

The Carnot's Cycle is derived, by taking into consideration thermodynamic parameters like Temperature, Pressure and Volume. The effects gravitational force (in other words acceleration force) on the processes of gases is ignored during the formulation of Carnot cycle in classical thermodynamics. The cycle that takes into consideration the effects of acceleration force along with all the parameters of classical thermodynamics is called "The Amin Cycle". Many interesting results are observed upon the inclusion of gravity (acceleration) during the formulation of a heat engine cycle.

According to classical thermodynamics the most efficient cycle is a constant entropy cycle. But analyzing the same cycle by the inclusion of the effects of acceleration indicates that the efficiency is further increased and a net decrease in entropy is attained. These results do not violate the second law of thermodynamics. They expand the domain of the second law to include the effects of acceleration. When the magnitude of acceleration is reduced to zero all the results reduce back to the original results of classical thermodynamics.

A typical example of a constant entropy cycle is the Carnot's Cycle. The Carnot Cycle can only convert heat to power if there is a temperature difference and the efficiency of the conversion of heat to power is proportional to the difference in temperatures.

Effects of acceleration are ignored in the formulation of the Carnot's cycle. When we consider the effects of acceleration and introduce it in a cycle, we have a cycle, which has an efficiency greater than the Carnot's cycle. When this advanced cycle also known as the Amin Cycle, is implemented in a practical configuration, we have a heat engine, which can convert heat into power with an acceleration difference.

An engine operating on the Amin Cycle compresses the working medium when it is accelerated and expands the working medium, simultaneously absorbing heat when the magnitude of its acceleration is lower.

There are several ways to achieve the variations in the magnitude of acceleration of the working medium. One method is as follows:

Let us consider a system in which two cylinders are placed on opposing ends of a centrifuge (a rotating cylinder with a constant radius). Figure 1 is a sketch of the proposed system. The centrifuge can vary in speed by changing the value of ω (reducing the rpm of the centrifuge). The cylinders are equipped with moving pistons located on the side closer to the center of the centrifuge. The pistons can compress and expand the cylinders. Now let us state the processes for the Amin cycle as applied to the system. The cycle comprises of the

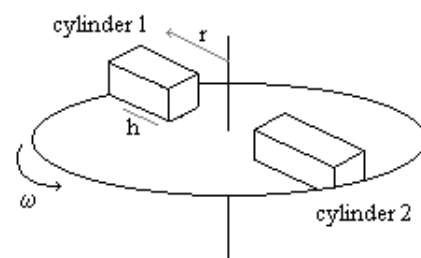


Figure 1

following steps (figure 2):

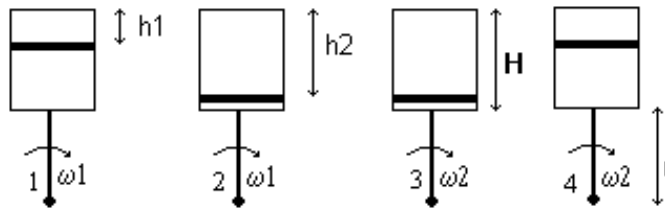


Figure 2

1. The gas expands isothermally at a temperature T in both cylinders at a low centrifuge speed ω_2 absorbing heat Q_1 and producing W_1 work.
2. The centrifuge increases in speed from ω_2 to ω_1 .
3. The gas is compressed isothermally at a temperature T in both cylinders at a high centrifuge speed ω_1 rejecting heat Q_3 and requiring W_3 work.
4. The centrifuge decreases in speed from ω_1 to ω_2 .

According to the First Law of Thermodynamics, energy is conserved during all the processes of the Amin Cycle. The first law of thermodynamics states that heat energy is equal to the addition of all the energies present in the system. The first law of thermodynamics is a statement that applies to any state whether simple or very complex.

$$dQ = dW + dU + dKE \quad (1)$$

All the processes in the Carnot's Cycle or any other conventional cycle are Quasi-Static Process. "A Quasi-Static Process, satisfies the following stringent requirements:

1. *Mechanical equilibrium.* There are no unbalanced forces acting on any part of the system or on the system as a whole.
2. *Thermal equilibrium.* There are no temperature differences between parts of the system and its surroundings.
3. *Chemical equilibrium.* There are no chemical reactions within the system and no motion of any chemical constituent from one part of a system to another part.

Once a system is in thermodynamic equilibrium and the surroundings are kept unchanged, no motion will take place and no work will be done. If, however, the sum of the external forces is changed so that there is a finite unbalanced force acting on the system, then the condition for mechanical equilibrium is no longer satisfied and the following situations may arise:

1. Unbalanced forces may be created within the system; as a result, turbulence, waves, etc., may be set up. Also, the system as a whole may execute some sort of accelerated motion.

2. As a result of this turbulence, acceleration, etc., a nonuniform temperature distribution may be brought about, as well as a finite difference of temperature between the system and its surroundings.
3. The sudden Change in the forces and in the temperature may produce a chemical reaction or the motion of a chemical constituent.”¹

If a process includes a kinetic energy difference, it implies that the system is in an acceleration field. The process in which the kinetic energy differences is greater than zero will fall under the definition of a non quasi-static process.

Free expansion is the main example of a non quasi-static process. In free expansion dU can be considered to be a function of T and V .

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad (2)$$

Since the process is isothermal, $dT = 0$ and $dU = 0$. It follows that $(\partial U/\partial V)_T = 0$, or in other words, U does not depend on V . Same analysis can be performed if U were dependent on P . It is apparent then that U is only a function of temperature.²

Experimental observation has confirmed that within the pressure range of 1 to 40 atm, it is seen that $(\partial u/\partial P)_\theta$ is independent of the pressure, depending only on the temperature.

The most extensive series of measurements of this kind was performed by Rossini and Frandsen in 1932 at the National Bureau of Standards with a method elaborated by Washburn. A bomb B contains n moles of gas at a pressure P and communicates with the atmosphere through a long coil wrapped around the bomb. The whole apparatus is immersed in a water bath whose temperature can be maintained constant at exactly the same value as that of the surrounding atmosphere.

The experiment is performed as follows: When the stopcock is opened slightly, the gas flows slowly through the long coil and out into the air. At the same time, the temperature of the gas, the bomb, the coils, and the water is maintained constant by an electric heating coil immersed in the water. The electrical energy supplied to the water is therefore the heat Q absorbed by the gas during the expansion. The work done by the gas is evidently

$$W = P_o(nv_o - V_B),$$

where P_o is atmospheric pressure, v_o is the molar volume at atmospheric temperature and pressure, and V_B is the volume of the bomb.

¹ Heat and Thermodynamics, page 52, 53

² Basic Engineering Thermodynamics, page 117

If $u(P, \theta)$ is the molar energy at pressure P and temperature θ and if $u(P_0, \theta)$ is the molar energy at atmospheric pressure and the same temperature, then, from the first law,

$$u(P, \theta) - u(P_0, \theta) = \underline{W - Q},$$

provided that corrections have been made to take account of the energy changes due to the contraction of the walls of the bomb. In this way, the energy change was measured for various values of the initial pressure and was plotted against the pressure. Since $u(P_0, \theta)$ is constant, it follows that the slope of the resulting curve at any value of P is equal to $(\partial u / \partial P)_\theta$. Within the pressure range of 1 to 40 atm, it is seen that $(\partial u / \partial P)_\theta$ is independent of the pressure, depending only on the temperature. Thus,

$$\left(\frac{\partial u}{\partial P}\right)_\theta = f(\theta),$$

and

$$u = f(\theta)P + F(\theta),$$

where $F(\theta)$ is another function of the temperature only.

Rossini and Frandsen's experiments with air, oxygen, and mixtures of oxygen and carbon dioxide lead to the conclusion that the internal energy of a gas is a function of both temperature and pressure. They found no pressure or temperature range in which the quantity $(\partial u / \partial P)_\theta$ was equal to zero.

Washburn's method has somewhat the same disadvantage as Joule's original method, in that the heat capacity of the gas is much smaller than that of the calorimeter and water bath. To keep the temperature of the gas constant within reasonable limits, the temperature of the water must be kept constant to within less than a thousandth of a degree. In Rossini and Frandsen's measurements, the final precision was estimated to be 2½ percent.

Another determination of the average value of $(\partial u / \partial P)_\theta$ of air in a pressure range of over 50 atm was made by Baker in 1983 with a somewhat different method, capable of an accuracy of 0.1 percent. In Baker's experiment, the air expanded from a thin spherical metal bomb into the space between it and a thin outer concentric spherical shell. The temperature of the outer shell was measured at small time intervals, starting from the moment after the expansion until the temperature returned to its original value; during this time, heat flowed into the gas at a rate that was a known function of the temperature difference between the outer shell and its surroundings, which were held at a constant temperature. By making the two shells very thin, the ratio of the heat capacity of the shells to that of the gas was very much smaller than in any previous work. The heat transferred to the gas could be calculated with great accuracy, and many corrections were applied with extreme care. The few measurements that have been made with this apparatus up to the present time have substantiated the results of Rossini and Frandsen.

ANALYSIS OF WORK:

The work done on the system can be determined by the pressure done on the surface of the piston multiplied by the volume displaced by the piston. If the pressure distribution were constant all over the cylinder then the work equation will be that of equation 3.

$$dW = PdV$$

$$dW = PAdh$$

$$W = NkTLn\left(\frac{h_2}{h_1}\right) \quad (3)$$

Since the pressure is not uniform and a pressure gradient exists due to the kinetic energy difference, the pressure at the piston is less than the pressure stated in equation 3. Therefore, if the displaced volume remains the same, the work done on the piston would be less than equation 3. Since we are considering an ideal gas, we can refer to the ideal gas equation:

$$P = \frac{NkT}{V} \quad (4)$$

where N is the number of molecules in the gas,
 k is Boltzmann's constant,
 T is the temperature of the gas,
 V is the volume of the gas.

When the gas is placed in a centrifuge, the molecules of the gas are subjected to a force that is equal to the mass of the molecule multiplied by the acceleration of the centrifuge. Since the process is isothermal, the temperature does not vary in the centrifuge. Consider the cylinder in the centrifuge as shown in Figure 3. To determine the pressure at a distance from the center of the centrifuge, we consider a volume with a constant area and a height Δr . The horizontal force at a unit area of distance r is the pressure P multiplied by the unit area A . The horizontal force at distance $r + dr$ has to equal the force at distance r plus the force due to the centrifuge of the gas in the unit section. Let ma be the force due to the centrifuge from each molecule of gas in the unit volume. The acceleration of the centrifuge $a = \omega^2 r$, where ω is the speed of the centrifuge, and r is the distance from the center of the centrifuge. Let N/V be the number of molecules per volume in the cylinder and Adr be the volume of the unit section. The total number of molecules in the unit section is $NAdr/V$. Therefore $AP_{r+dr} - AP_r = AdP = Nm\omega^2 rAdr/V$. Now from the equation of ideal gas, $P = NkT/V$. Since N and P are directly related we can solve for P or N , say P .

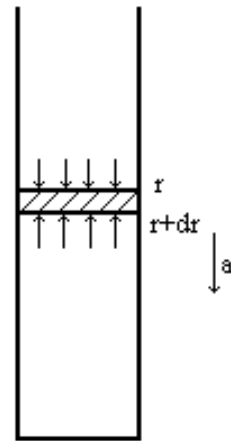


Figure 3.

$$dP = \frac{N}{V} m \omega^2 r dr$$

$$dP = P \frac{m \omega^2}{kT} r dr \quad (5)$$

where m is the mass of a molecule of the gas,
 ω is the speed of the centrifuge,
 k is Boltzmann's constant,
 T is the Temperature of the gas,
 r is the distance from the center of the centrifuge,
 P is the pressure of the gas.

If the speed of the centrifuge were zero, then the pressure differential would be zero and equation 4 would be used to calculate the pressure at any point. The distribution of the pressures in the system does not alter the value of the average pressure of the gas. Therefore, we can define a layer in the system where the pressure at that layer is equal to the average pressure, or the pressure with out the centrifugal affect. Let us call the pressure at that layer P_c . The layer is located at a distance r_c from the center of the centrifuge. The values of P_c and r_c change as the volume change. The value of P_c can be determined by using the ideal gas equation 4, and we can solve for the value of r_c . Integrating equation 5 from the layer at distance r_c to any layer at distance r yields:

$$\int_{P_c}^P \frac{dP}{P} = \int_{r_c}^r \frac{m \omega^2}{kT} r dr$$

$$P = P_c e^{\frac{1}{2} \frac{m \omega^2 (r^2 - r_c^2)}{kT}}$$

There are several ways to calculate the work done in a process where the kinetic energy difference is not zero. One of the methods to calculate the work done is to determine the reduction in energy due to the kinetic energy difference and subtract that from the work done according to equation 3. The energy analysis for **process 1** is as follows:

$$dW' = dW - dKE$$

$$W' = NkTLn\left(\frac{h_2}{h_1}\right) - \left(\frac{1}{2} Nm \omega_2^2 r_{c1}^2 - \frac{1}{2} Nm \omega_2^2 r_{c2}^2\right) \quad (6)$$

where r_{c1} is the distance from the centrifuge to the average height h_1 in the piston.
 r_{c2} is the distance from the centrifuge to the average height h_2 in the piston.

We can solve for r_{c1} and r_{c2} by comparing the potential energy difference to the kinetic energy difference.

$$\begin{aligned}
 dPE &= dKE \\
 Nma((r+H)-(r+H-h)) &= \frac{1}{2}Nm\omega^2(r+H)^2 - \frac{1}{2}Nm\omega^2(r+H-h)^2 \\
 Nm\omega^2 r_c h &= \frac{1}{2}Nm\omega^2((r+H)^2 - (r+H-h)^2) \\
 r_c &= \frac{1}{2} \frac{(2(r+H)h - h^2)}{h} \\
 r_c &= (r+H) - \frac{1}{2}h \tag{7}
 \end{aligned}$$

Substituting the value of r_c into equation (6):

$$\begin{aligned}
 W' &= NkTLn\left(\frac{h_2}{h_1}\right) - \frac{1}{2}Nm\omega_2^2\left(r+H-\frac{1}{2}h_1\right)^2 + \frac{1}{2}Nm\omega_2^2\left(r+H-\frac{1}{2}h_2\right)^2 \\
 W' &= NkTLn\left(\frac{h_2}{h_1}\right) - NkT\frac{1}{2}\frac{m\omega_2^2}{kT}\left(r+H-\frac{1}{2}h_1\right)^2 + NkT\frac{1}{2}\frac{m\omega_2^2}{kT}\left(r+H-\frac{1}{2}h_2\right)^2 \\
 W' &= NkTLn\left(\frac{h_2}{h_1}\right) - NkTLn\left(e^{\frac{1}{2}\frac{m\omega_2^2}{kT}\left(r+H-\frac{1}{2}h_1\right)^2}\right) + NkTLn\left(e^{\frac{1}{2}\frac{m\omega_2^2}{kT}\left(r+H-\frac{1}{2}h_2\right)^2}\right) \\
 W' &= NkTLn\left(\frac{h_2}{h_1} \frac{e^{\frac{1}{2}\frac{m\omega_2^2}{kT}\left(r+H-\frac{1}{2}h_2\right)^2}}{e^{\frac{1}{2}\frac{m\omega_2^2}{kT}\left(r+H-\frac{1}{2}h_1\right)^2}}\right) \tag{8}
 \end{aligned}$$

Equation 8 represents the work done by expansion at a low centrifugal speed ω_2 . If $\omega_2 = 0$, the equation 8 would simplify back into equation 3. The kinetic energy difference, dKE for the expansion, can be calculated as follows:

$$dKE = \frac{1}{2}m\omega_2^2 r_{c1}^2 - \frac{1}{2}m\omega_2^2 r_{c2}^2 \tag{9}$$

From equation 1, the first law of thermodynamics, $dQ = dU + dW' + dKE$.

$$\begin{aligned}
dQ &= dU + dW' + dKE \\
dQ &= 0 + dW - dKE + dKE \\
Q_1 &= W_1 = NkTLn\left(\frac{h_2}{h_1}\right)
\end{aligned} \tag{10}$$

Therefore, the heat required during expansion in the Amin Cycle is the same as the heat required during expansion in any other known cycle. The work done during expansion is less than the heat required for expansion.

In **process 2** the volume of the cylinder remains constant but the speed varies from ω_2 to ω_1 . Since no volume change takes place, $dW = dW' = 0$. We have already mentioned that the process is isothermal which implies $dU = 0$. Let us analyze the energy equation from equation 1.

$$\begin{aligned}
dQ &= dU + dW' + dKE \\
dQ &= 0 + 0 + dKE \\
Q_2 &= \frac{1}{2} Nm\omega_1^2 r_{c1}^2 - \frac{1}{2} Nm\omega_2^2 r_{c1}^2 \\
Q_2 &= \frac{1}{2} Nm\omega_1^2 \left(r + H - \frac{1}{2} h_1\right)^2 - \frac{1}{2} Nm\omega_2^2 \left(r + H - \frac{1}{2} h_1\right)^2 \\
Q_2 &= \frac{1}{2} Nm(\omega_1^2 - \omega_2^2) \left(r + H - \frac{1}{2} h_1\right)^2
\end{aligned} \tag{11}$$

Process 3 is the compression from cylinder height h_1 to cylinder height h_2 at centrifuge speed of ω_1 . The analysis of the compression process is identical to the expansion cycle. Thus the derivation done for the work of compression would be identical with the exception of the change of parameters to include the speed ω_1 .

$$W_3 = NkTLn\left(\frac{h_1 e^{\frac{1}{2} \frac{m\omega_1^2}{kT} \left(r+H-\frac{1}{2}h_1\right)^2}}{h_2 e^{\frac{1}{2} \frac{m\omega_1^2}{kT} \left(r+H-\frac{1}{2}h_2\right)^2}}\right) \tag{12}$$

The heat analysis for the compression process is also identical to the expansion heat derivation.

$$\begin{aligned}
dQ &= dU + dW' + dKE \\
dQ &= 0 + dW - dKE + dKE \\
Q_3 &= W_3 = NkTLn\left(\frac{h_1}{h_2}\right)
\end{aligned} \tag{13}$$

Therefore, the heat required during compression in the Amin Cycle is the same as the heat required during compression in any other known cycle. The work done during compression is less than the heat required for compression.

In **process 4** the volume of the cylinder remains constant but the speed varies from ω_2 to ω_1 . Since no volume change takes place, $dW = dW' = 0$. We have already mentioned that the process is isothermal which implies $dU = 0$. Let us analyze the energy equation from equation 1.

$$\begin{aligned}
 dQ &= dU + dW' + dKE \\
 dQ &= 0 + 0 + dKE \\
 Q_4 &= \frac{1}{2} Nm\omega_1^2 r_{c1}^2 - \frac{1}{2} Nm\omega_2^2 r_{c1}^2 \\
 Q_4 &= \frac{1}{2} Nm\omega_1^2 \left(r + H - \frac{1}{2} h_1 \right)^2 - \frac{1}{2} Nm\omega_2^2 \left(r + H - \frac{1}{2} h_1 \right)^2 \\
 Q_4 &= \frac{1}{2} Nm(\omega_1^2 - \omega_2^2) \left(r + H - \frac{1}{2} h_1 \right)^2 \tag{14}
 \end{aligned}$$

Let us now examine the net work done by or on the system by adding the work at each stage. The net work done is the work due to expansion plus the work due to gravity change plus the work done due to compression plus the work done due to gravity change.

$$\begin{aligned}
 Net_Work &= W_1 + W_2 + W_3 + W_4 \\
 Net_Work &= NkTLn \left(\frac{h_2 e^{\frac{1}{2} \frac{m\omega_2^2}{kT} \left(r + H - \frac{1}{2} h_2 \right)^2}}{h_1 e^{\frac{1}{2} \frac{m\omega_2^2}{kT} \left(r + H - \frac{1}{2} h_1 \right)^2}} \right) + 0 + NkTLn \left(\frac{h_1 e^{\frac{1}{2} \frac{m\omega_1^2}{kT} \left(r + H - \frac{1}{2} h_1 \right)^2}}{h_2 e^{\frac{1}{2} \frac{m\omega_1^2}{kT} \left(r + H - \frac{1}{2} h_2 \right)^2}} \right) + 0 \\
 Net_Work &= NkTLn \left(\frac{e^{\frac{1}{2} \frac{m}{kT} (\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_2 \right)^2}}{e^{\frac{1}{2} \frac{m}{kT} (\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_1 \right)^2}} \right) \\
 Net_Work &= NkTLn \left(e^{\frac{1}{2} \frac{m}{kT} (\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_2 \right)^2} \right) \\
 &- NkTLn \left(e^{\frac{1}{2} \frac{m}{kT} (\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_1 \right)^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 Net_Work &= \frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_2 \right)^2 \\
 &- \frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_1 \right)^2 \\
 Net_Work &= \frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(\left(r + H - \frac{1}{2} h_2 \right)^2 - \left(r + H - \frac{1}{2} h_1 \right)^2 \right) \quad (15)
 \end{aligned}$$

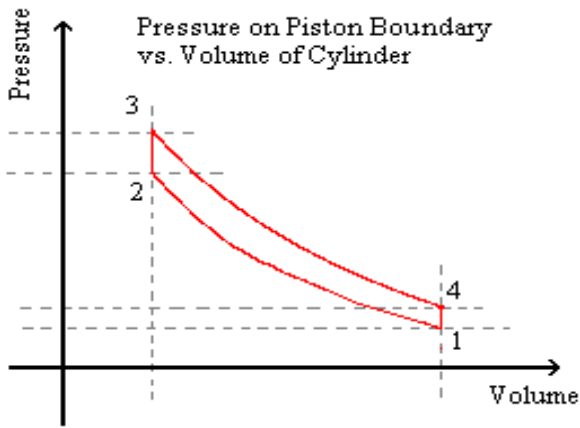


Figure 4, is a Pressure – Volume diagram for the Amin Cycle. Process 1-2 is compression of working medium at higher magnitude of kinetic energy, Process 2-3 is the decrease in kinetic energy of the working medium at constant volume, Process 3-4 is the expansion of the working medium at lower magnitude of kinetic energy and Process 4-1 is the increase in kinetic energy of the working medium at constant volume. The area 1-2-3-4 is the net heat converted to power.

Let us also look at the heat absorbed or rejected by the system for the entire cycle.

$$\begin{aligned}
 Total_Heat &= Q_1 + Q_2 + Q_3 + Q_4 \\
 Total_Heat &= NkTLn\left(\frac{h_2}{h_1}\right) + \frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_2 \right)^2 \\
 &- NkTLn\left(\frac{h_2}{h_1}\right) - \frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_1 \right)^2 \\
 Total_Heat &= \frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(\left(r + H - \frac{1}{2} h_2 \right)^2 - \left(r + H - \frac{1}{2} h_1 \right)^2 \right) \quad (16)
 \end{aligned}$$

The first law of thermodynamics states that the energy for the entire cycle is conserved. Equations (15) and (16) are equal and thus energy is conserved. Thus the Amin Cycle converts heat into work without a temperature differential but with only a kinetic energy differential.

The main difference between the Amin Cycle and the Carnot's Cycle is that in the Amin Cycle, the compression and expansion processes are done at different magnitudes of Kinetic Energy, while in the Carnot's Cycle, the processes are done at the same magnitude of Kinetic Energy.

The efficiency is:

$$\varepsilon = \frac{Net_Work}{Heat_In}$$

$$\varepsilon = \frac{\frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(\left(r + H - \frac{1}{2} h_2 \right)^2 - \left(r + H - \frac{1}{2} h_1 \right)^2 \right)}{NkTLn\left(\frac{h_2}{h_1}\right) + \frac{1}{2} Nm(\omega_2^2 - \omega_1^2) \left(r + H - \frac{1}{2} h_2 \right)^2} \quad (17)$$

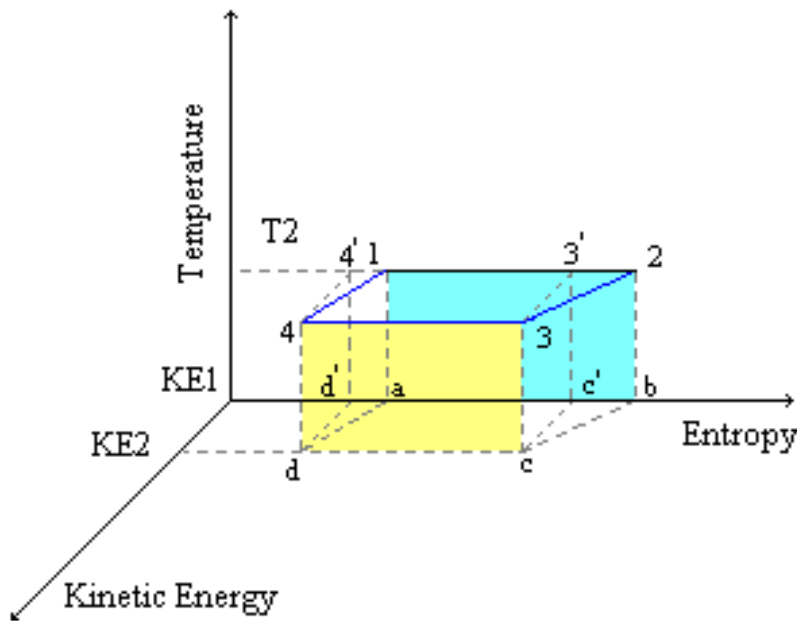


Figure 5, is the Temperature-Entropy Diagram, a true representation of the Amin Cycle. The addition of the third dimension, the kinetic energy, better describes the increase and decrease in kinetic energy with respect to entropy. The points 3' and 4', which are points 3 and 4 in figure (a), are the projection of points 3 and 4 respectively on the Temperature-Entropy plane. The area (1-2-b-a) is the heat input, and the area (4-3-c-d) is the heat output. KE2 is the lower kinetic energy value and KE1 is the higher kinetic energy value. Area (4-3-c-d) is smaller than Area (1-2-b-a) and the difference between these two areas is the heat converted to power. The work done to increase and decrease the kinetic energy does not depend on the path but only on the end points.

Summary of Cycle

Processes:

- 1 – 2 : Expansion of cylinder from h_1 to h_2 at gravity g_2 .
 2 – 3 : Change of gravity from g_2 to g_1 at cylinder height h_2 .
 3 – 4 : Compression of cylinder from h_2 to h_1 at gravity g_1 .
 4 – 1 : Change of gravity from g_1 to g_2 at cylinder height h_1 .

Input Parameters:

- h_1 = height of cylinder in compressed stage.
 h_2 = height of cylinder in expanded stage.
 g_1 = high gravity field in which compression is done.
 g_2 = low gravity field in which expansion is done.
 T = temperature of the gas during the isothermal processes.
 k = Boltzman's constant.
 m = mass of an air molecule.
 N = total number of molecules in the cylinder.
 A = area of the cylinder.

Derived Parameters:

- P_c = pressure layer in the cylinder that is equal to the pressure given by the ideal gas law.
 h_c = height from the bottom of the cylinder to the layer defined above.
 P_0 = Pressure at the bottom of the cylinder.
 P_h = Pressure at the piston surface.
 W = Work of each process.
 PE = Potential energy of each process.
 Q = Heat energy of each process.

Fundamental Equations:

$$\begin{aligned} dQ &= dU + dW + dPE && \text{Conservation of Energy} && (1) \\ dW &= APdh && \text{Work Equation} && (2) \\ P &= NkT/V && \text{Ideal gas law} && (3) \end{aligned}$$

Derived Equations:

$$P = \frac{Nmg}{A} \frac{e^{-\frac{mgh}{kT}}}{1 - e^{-\frac{mgh}{kT}}} \quad \text{General pressure derivation which depends on the height.} \quad (5)$$

$$h_c = \frac{kT}{mg} \ln \left(\frac{mg}{kT} h \left(\frac{1}{1 - e^{-\frac{mgh}{kT}}} \right) \right) \quad \text{Height as defined in the derived parameters.} \quad (10)$$

Process Equations:

$$(1-2) \quad dQ = dW + dPE \quad \Delta Q_{1-2} = NKTLn\left(\frac{h_2}{h_1}\right)$$

$$\Delta PE_{1-2} = NKTLn\left(\frac{h_2}{h_1} \left(\frac{1 - e^{-\frac{mg_2 h_1}{kT}}}{1 - e^{-\frac{mg_2 h_2}{kT}}}\right)\right) \quad W_{1-2} = NKTLn\left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}}\right)$$

(2-3)

$$\Delta PE_{2-3} = NKTLn\left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_2}{kT}}}\right)\right) \quad \Delta Q_{2-3} = NKTLn\left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_2}{kT}}}\right)\right)$$

(3-4)

$$dQ = dW + dPE$$

$$W_{3-4} = -NKTLn\left(\frac{1 - e^{-\frac{mg_1 h_2}{kT}}}{1 - e^{-\frac{mg_1 h_1}{kT}}}\right) \quad \Delta PE_{3-4} = -NKTLn\left(\frac{h_2}{h_1} \left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_1 h_2}{kT}}}\right)\right)$$

(4-1)

$$\Delta Q_{3-4} = -NKTLn\left(\frac{h_2}{h_1}\right)$$

$$\Delta PE_{4-1} = -NKTLn\left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}}\right)\right) \quad \Delta Q_{4-1} = -NKTLn\left(\frac{g_2}{g_1} \left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}}\right)\right)$$

Results:

$$\Delta PE_{Total} = \Delta PE_{12} + \Delta PE_{23} + \Delta PE_{34} + \Delta PE_{41} = 0$$

$$\Delta W_{Total} = \Delta W_{12} + \Delta W_{34}$$

$$\Delta Q_{Total} = Q_{12} + Q_{23}$$

$$\Delta W_{Total} = NKTLn\left(\left(\frac{1 - e^{-\frac{mg_2 h_2}{kT}}}{1 - e^{-\frac{mg_2 h_1}{kT}}}\right)\left(\frac{1 - e^{-\frac{mg_1 h_1}{kT}}}{1 - e^{-\frac{mg_1 h_2}{kT}}}\right)\right)$$